# Choosing the Optimal Parameters of the Gaussian Filter for Obtaining the Surface Relief Components

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Abstract—It is known that for the determination of mechanical, corrosive and tribological parameters, such terms as "roughness" and "waviness" are often used. Filtering in the frequency domain is used to extract these components from the total relief. In this paper, we propose an approach for determining the optimum value of the cut-off frequency for 2D Gaussian filter, for our method of retrieving the surface relief by three-step phase-shifting interferometry with an arbitrary phase shift of the reference beam.

*Index Terms*—2D filtering; interferometry; surface relief; roughness; waviness.

### I. INTRODUCTION

Investigation of the surface relief of the material surface is an actual task for the control and evaluation of the quality of the assemblies under the static and cyclic loads or in an aggressive media. At the same time, informative parameters can be both surface macro- and micro-relief. Various methods are used for retrieving the total relief, which can be divided into contact and non-contact ones. Among the non-contact methods, the most widely used today are SEM [1], digital holography [2], and phase-shifting interferometry (PSI) [3]. We proposed a new approach for the reconstruction the surface relief using the three-step phase-shifting interferometry method with an arbitrary phase shift of the reference beam [4]. This method allows the extraction of surface relief components using two sequential iterations with filtration in the frequency domain (FFD).

At the stage of separation the roughness and waviness components, there is a problem of choosing the optimal filters and their parameters. In the three-step method, the problem is complicated, since the filtration procedure must be applied twice. Therefore, in this article, we consider the application of 2D Gaussian filter to separate the components of a total surface Kotsiuba Yurii

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relief by the three-step PSI method. By retrieving the surface relief components from the simulated test phase map, and comparing them to initial ones, the optimal parameters for this filter are selected and an approach that can be used for other 2D filters is proposed.

## II. MATHEMATICAL DESCRIPTION OF THE TREE-STEP PSI METHOD

The essence of the three-step phase-shifting interferometry method consists in sequentially recording three digital interferograms  $I_n(x,y)$  with an arbitrary phase shift of the reference beam. The phase map of the surface relief is obtained using the following relation:

$$\varphi(x, y) = \arctan\left(-\frac{c}{b}\right) \tag{1},$$

where:

$$b = [I_1(x, y) - I_3(x, y)] \sin \alpha_{21} + [I_2(x, y) - I_1(x, y)] \sin \alpha_{31}$$
(2)

$$c = [I_2(x,y) - I_3(x,y)] + [I_3(x,y) - I_1(x,y)] \cos \alpha_{21} + [I_1(x,y) - I_2(x,y)] \cos \alpha_{31}$$
(3)

The value of the phase shift  $\alpha_{nl}$  between two interferograms is obtained from:

$$\alpha_{n1} = \arccos \frac{\left\langle \left[ I_1(x,y) - \left\langle I_1(x,y) \right\rangle \right] \left[ I_n(x,y) - \left\langle I_n(x,y) \right\rangle \right] \right\rangle}{\sigma_{I_1(x,y)} \sigma_{I_n(x,y)}}$$
(4),

where  $\sigma$  denotes standard deviation.

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The obtained surface phase map contains information about the roughness and waviness of the object. After that, the surface relief components are obtained using the FFD with two iterations according to the algorithm given in [4]. It should be noted that at the first iteration, the FFD is applied to the continuous *cos* and *sin* components of the phase map. And at the second iteration performing the FFD of the surface relief obtained at the previous iteration, the components are separated. Moreover, the roughness is calculated by subtracting the resulting waviness component from the initial surface relief.

Obviously, that during retrieving the relief components by this method, it is important to choose the optimum cut-off frequency  $f_c$  at each iteration step.

#### III. DESCRIPTION OF CONSIDERED 2D FILTER

In general case the FFD of initial image F(x,y) can be described by the following equation:

$$F'(x, y) = FFT^{-1} \{ G(x, y) \cdot FFT \{ F(x, y) \} \}$$
(5),

where  $FFT\{\ldots\}$  denotes fast Fourier transform; G(x, y) represents the transfer function.

In this paper, it was proposed to use the Gaussian filter to separate the surface relief components, due to its popularity and simple implementation. In the general case, for a twodimensional case, it is described by the following relation:

$$G_{Gauss}\left(x,y\right) = \exp\left[-\frac{u^2 + v^2}{D_0^2}\right]$$
(6),

where u, v are the image coordinates in the frequency domain,

$$D_0 = \sqrt{u_c^2 + v_c^2}$$
(7)

The 3D representation of this filter is shown in Fig.1.



Figure 1. General view of the 2D Gaussian filter

Using the proposed filter a technique for determination the optimal values of the cut-off frequency was developed.

### IV. DESCRIPTION OF THE TECHNIQUE FOR DETERMINATION THE OPTIMAL CUT-OFF FREQUENCY

For implementation our determination method a test surface was created. It contains the waviness w(x,y) and roughness r(x,y) components. The first component was obtained using standard function "peaks" with dimension 500×500 pixels. The

roughness component was obtained by adding Gaussian noise with a value  $\pm 0.065 \ \mu\text{m}$ . The general view of the test surface is shown in Fig.2.



Figure 2. Test surface

After that the phase map of the test surface (see Fig.3.) was obtained according to the relation:

$$\varphi(x, y) = \arctan\left(\frac{\operatorname{Im}\left(\exp\left[iT(x, y)\right]\right)}{\operatorname{Re}\left(\exp\left[iT(x, y)\right]\right)}\right)$$
(8),

where T(x,y) stands for the total relief, which consist of roughness and waviness components.



Figure 3. Phase map of the test surface

Using the obtained phase map, the relief components of the test surface were separated by the double iteration algorithm described in the previous section. For estimating the errors, the standard deviation between the initial relief components and those obtained by our algorithm was considered. At each iteration step, the value of  $f_c$  for the Gaussian filter was set within the range 0.0012- $0.021 \mu m^{-1}$ .

### V. RESULTS AND DISCUSSION

As a result, it was estimated that the error value is rather small and varies little practically within the entire proposed frequency range. Starting from  $0.0028\mu m^{-1}$  a rapid error increase is observed. However, for the first iteration step, the smallest value of the error for both micro- and macro-relief is achieved at a cut-off frequency close to this region  $(f_{cl} = 0.0036 \ \mu m^{-1})$ .

A quite different situation is observed when choosing the optimum cut-off frequency for the second iteration step. The features of the interconnection between the cut-off frequencies on two iteration steps are shown in the graph in Fig.4. Here the cut-off frequencies on the first iteration step (x-axis), and the appropriate cut-off frequencies on the second iteration step, for which the minimum error is achieved, are shown.



Figure 4. Interconection between cut-off frequencies on first and second iteration step

From the following graph it is seen that on the second iteration step almost for any values of  $f_{c1}$  optimal cut-off frequencies for the roughness and waviness components do not match, and are equal  $f_{c2} = 0.0136\mu m^{-1}$  and  $f_{c2} = 0.009\mu m^{-1}$ , respectively. The intersection of two curves is located at point where the standard deviation significantly increases.

However, the proposed approach with standard deviation does not provide the full information about separation errors. During the determination of the relief components with decreasing the  $f_{c2}$  some distortion of the micro-relief occurs. In this case, some part of macro-relief component appears in micro-relief. In the frequency domain, this feature causes increasing of the central peak height. Thus, two Fourier spectrums of the separated roughness components (using two obtained values of  $f_{c2}$ ) were calculated (see Fig.5.).



Figure 5. Spectrum of the roughness component obtained with  $f_{c2} = 0.0136\mu m^{-1}$  (a) and  $f_{c2} = 0.009\mu m^{-1}$  (b)

From the Fig.5. one can observe that despite at  $0.009\mu m^{-1}$  minimum standard deviation value is provided, the overall image of micro-relief is disturbed due to the presence of macro-relief part. That's why, standard deviation of the roughness component cannot be used as the main criterion during the determination of the optimal cut-off frequency on the second iteration step.

### CONCLUSION

The technique for determination the optimal values of the cut-off frequencies was developed and proposed for three-step phase shifting interferometry. Optimal cut-off frequencies for roughness and waviness components of the test surface were determined using least-error approach. Also, the results have shown that for the 2D Gaussian filter standard deviation of micro-relief could not provide optimal cut-off frequency determination due to the distortion on the macro level. This approach will be used for other 2D filters to improve the understanding of the relief components separation process.

### REFERENCES

- Garcia, N., Baró, A. M., Miranda, R., Rohrer, H., Gerber, C., Cantu, R. G., & Pena, J. L. (1985). Surface roughness standards, obtained with the scanning tunneling microscope operated at atmospheric air pressure. Metrologia, 21(3), 135.
- [2] Picart, P., & Li, J. C. (2013). Digital holography. John Wiley & Sons.
- [3] Schreiber, H., & Bruning, J. H. (2007). Phase shifting interferometry. Optical shop testing, 547-666.
- [4] Muravsky, L. I., Kmet, A. B., Stasyshyn, I. V., Voronyak, T. I., & Bobitski, Y. V. (2018). Three-step interferometric method with blind phase shifts by use of interframe correlation between interferograms. Optics and Lasers in Engineering, 105, 27-34.