The Optico-Geometrical Analysis of the Planar Waveguides and Modulators

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Abstract—**In this study, the analysis of transverse resonance and modulations effects in an optico-geometrical description of waveguides, is considered. The expression for calculation of the propagation angle of waveguide modes and verios parameter of modulators are programmed. Calculations are done for symmetric waveguides made of gallium arsenide and lithium niobate . Apart from different refraction indices, the programme allows to register waveguide thickness, mode order and radiant wave length. The programme is created on C++ and C# in the *Embarcadero* and MS Visual Studio 17 programming environment.**

Index Terms—**optical veguides, gallium arsenide, lithium niobate,electro-optical modulators**

I. Introduction

One of the important tasks of modern optoelectronics nowadays is to substitute electronic integrated circuits for equivalent ones and, perhaps, for more efficient integrated optical circuits [1-5]. That is why integrated optical components should be space-saving and miniature, reliable, with high mechanic and thermal stability, low power intake and should undergo integration on one underlay, or "chip" [1,2]. In several studies, based on integrated optical elements, the possibility of creating optical computers [3-5], which considerably exceed existing radio-technical computing machines due to their parameters, is considered. The analysis of literary data showed that optical computers first of all demand to create passive elements with pre-determined characteristics. On the one hand, these demands stimulate the development of improved methods of thin film production. On the other hand, they stimulate the modelling and estimation of film characteristics. In the majority of cases, while considering appliances of integrated optics, the theoretical model is used. It is based on electromagnetic theory [1, 2]. description of light propagation in a flat waveguide from the point of view of geometrical optics [1-5].

The view of the flat waveguide is presented in Fig.1.

The methods of geometrical optics regarding flat waveguides were scrutinised in several studies [1-6]. But in many cases, they were general, so it was impossible to use them for practical purposes. New opportunities of waveguide modelling created the achievement of computer technology. In this study, the programme created in the *Embarcadero*® RAD Studio XE3 programming environment for analysis of planar dielectric waveguides is presented.

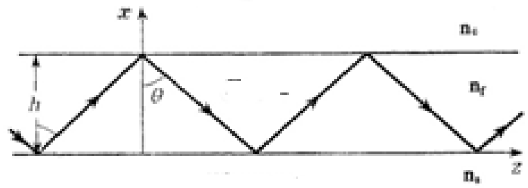

Fig.1. Light propagations in the flat dielectric waveguide

From the point of view of geometrical optics, light propagation in a flat dielectric waveguide can be presented as ray propagation. As a result of full internal reflection from a boundary between the media of a waveguide-underlay and waveguide-top coat, the ray moves in a zigzag path. In such a case, the phenomenon of reflection and refraction at the boundary of two dielectrics plays a key role. We will pay attention to TE mode. It means that vectors of intensity of electric fields are perpendicular to an incident plane, in which normal to wave surface and to the boundary of media lie. In this case, the expression for reflection coefficient can be written as follows:

$$R_{TE} = (n_1 \cos\theta_1 - n_2 \cos\theta_2)/(n_1 \cos\theta_1 + n_2 \cos\theta_2) = \\ = (n_1 \cos\theta_1 - \sqrt{n_2^2 - n_1^2 \sin^2\theta_1})/(n_1 \cos\theta_1 + \sqrt{n_2^2 - n_1^2 \sin^2\theta_1}) \quad (1)$$

where n_1 and n_2 are the indices of the refraction of media, in which beam incidence and refraction occur respectively. θ_1 and θ_2 are the angles of incidence and refraction respectively.

Critical angle θ_c is found by means of the following expression:

$$\sin\theta_c = n_2 / n_1 \quad (2)$$

While in equation $\theta_1 < \theta_c$ is fulfilled, we have only partial reflection of light, and R takes conventional true value.

As soon as angle of incidence θ_m is greater than value θ_c $\sin\theta_c = n_2 / n_1$, modulus of reflection coefficient $|R| = 1$, and it is possible to talk about the total light reflection. In this case, R is complex, and the reflected beam has a phase shift with respect to the incident one by angle φ_{TE}. Then, we can put down the following expression:

$$R = \exp(2/\varphi_{TE}) \tag{3}$$

and get the expression of phase shifts φ_{TE} by means of Fresnel formula

$$tg\varphi_{TE} = \sqrt{n_1^2 \sin^2\theta_1 - n_2^2} / (n_1 \cos\theta_1) \tag{4}$$

Beam propagation in the waveguide is described by the self-consistency condition, which is also called the condition of transverse resonance:

$$2kn_f h\cos\theta - 2\varphi_s - 2\varphi_c = 2\upsilon\pi \tag{5}$$

where υ is a whole number (0,1,2…), which defines the mode order, phase shifts φ_s i φ_2 are the functions of incident angle θ. Equity (6) is actually a dispersion equation of the waveguide, which defines constant of propagation β as function of frequency ω and film thickness h:

$$\beta = \omega / \upsilon_p = kn_f \sin\theta \tag{6}$$

Now we will put down dependency (5) for the case of a symmetric waveguide, when $\varphi_s = \varphi_c$:

$$kn_f h\cos\theta = 2\varphi_s + \upsilon\pi \tag{7}$$

As long as equation (7) is transcendental, the best way to solve it is to use the graphic method. The intersection of right and left parts of equation (7) shows angle value θ for zigzag wave of order ν.

Planar waveguides is used for creation optical modulators. One of the possible structures of such planar modulator is presented on Fig.2

The reasons for interest to such devises is deal firstly – with the possibility of reducing in the stripped waveguides the parameters b and g to the magnitude of wavelength, and secondly – the possibility of reducing required electrical capacity.

Present method of modulation is used two unimodes stripped waveguides of the electrooptical materials that where implicated on the common substrate. Both waveguides are parallel one to each other's and divided by gap g. The length of the area where they are parallel is equals l. Outside of this area the distance between waveguides much larger then g. The electrodes are situated such as it can be showed in the Fig.2. When the electrical voltage is applied to the external electrodes, then the boundary electric field in the waveguides 1 has the opposite sight then in the waveguides 2. When the waveguides 1 and 2 are the same, and the conditions of phase adjustments $\beta_1 = \beta_2$ take place, then the light stream from the waveguides 1 will be to pump up to the waveguides 2, and contrary. Another conditions of this process is little amount of parameter *g*. It is necessary for overlapping of fading electrical field of both waveguides.

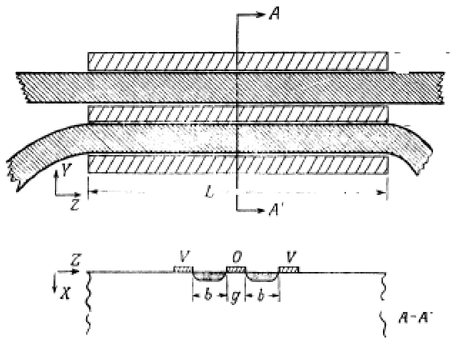

Fig.2 Appearance of the electrooptical planar modulator with connected dielectric waveguides

The distance on which the light is pumping totally from waveguides 1 to waveguides 2 in the conditions of phase adjustments is called connection length and determined as

$$L = \pi / 2\aleph \tag{8}$$

where $\aleph$ –is so called constant of connections.

When the conditions of phase adjustments is not fulfilled i.e.

$$\Delta = [\beta_1 - \beta_2] \neq 0, \tag{9}$$

then only part of light will be pumped from one waveguides to another and period of the connections become shorter.

When the initial intensity of light in the waveguide 1 is I_1, and initial intensity of light in the waveguide 2 is I_2=0, then the ratio of the intensities is defined by expression [7]:

$$\left(\frac{I_1}{I_2}\right) = \left[\aleph^2 \Big/ (\aleph^2 + \Delta^2)\right] \sin^2\left[\left(\aleph^2 + \Delta^2\right)^{1/3} z\right] \tag{10}$$

In the condition: $\Delta^2 >> \aleph^2$, the bound between waveguides is absent.

Let us suppose that thickness of the waveguides picked up so that TE_0-mode will be propagating but for the TE_1-mode take place clipping conditions. This supposing allows describe:

$$\delta = \left(\frac{1}{\pi}\right)\left[\left(2\sqrt{2n_s}\Delta n \Big/ \lambda_0 \right)^2 - \left(a + \lambda_0 \Big/ \pi\sqrt{2n_s}\Delta n\right)^{-2}\right]^{1/2} \tag{11}$$

where

$$\beta = \left(\frac{2\pi n}{\lambda_0}\right)\sqrt{2\left(\Delta n - \delta n\right)} / n_s + 1,$$

$$\Delta n = n_f - n_s = \beta/k - n_s, k = 2\pi/\lambda_0, k_2 = \frac{\pi}{\left(\beta + \lambda_0 / \pi\sqrt{2n_s}\Delta n\right)},$$

Electrooptical modulations in such case are based on the changing of Δ amount. Easy to shown that:

$$\Delta = \pi r'(n')^3 \frac{2E_y}{2\lambda}, \qquad (13)$$

where E_y - is applied electrical field; *r*', *n*' – effective electrooptical coefficient and effective refractive index accordingly.

II. Results

The programme is created on C++ and C# in the *Embarcadero* and MS visual studio- 17 programming environment. By means of a keyboard the values of wavelength λ, index of waveguiding layer refraction n_f, mode order υ and waveguide thickness *h* are assigned.

The characteristics of modelling symmetric waveguides made of gallium arsenide are calculated in this work to serve as examples. It is worth noting that gallium arsenide is the only chemical agent nowadays on which all the functions of optical elements are based. These are light generation, its propagation through the waveguide, input of emission and its output from the waveguide, modulation and detection.

As it follows from equation (7), there is no condition of cut-off for zero mode. It means that it will propagate in the waveguide whatever its shallow thickness is. When increasing mode order, not all waves can propagate through the waveguide. In picture 1 the dependency of transcendental equation solution for zero mode $\upsilon = 0$ of a semiconducting symmetric waveguide made of gallium arsenide is presented.

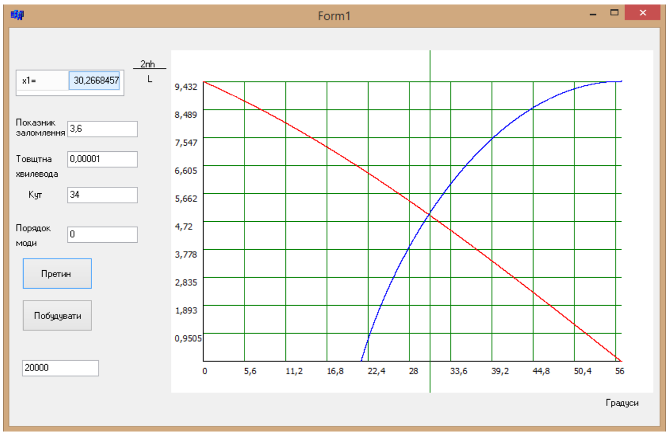

Fig.3. Graphic solution of dispersive for zero TE mode symmetric waveguide made of gallium arsenide.

Judging from this picture, the angle at which waveguide mode propagates corresponds to the point of intersection of two curves: $kn_f h\cos\theta$ and $2(\varphi_s + \pi)$ is equal to 30,266°. Moreover, when increasing relationship h/λ, the period of zigzag wave will reduce (angle θ is smaller), but the solution always exists even if film thickness becomes very shallow. This means that in such a waveguide there is no condition of cut-off for main mode.

Based on the result, it is possible to calculate several parameters, which have practical use. According to the idea of zigzag waves, constant of propagation β for waveguiding mode in a flat waveguide (and phase velocity υ_p, which is connected to it) is found by means of expression (6) and is a *z*-component of wave vector kn_f.

Judging from Picture 1, it is also understandable that the created programme and graphic realisation of the results allows calculating the angle of reflection for TE modes of random order, the thickness of waveguides and their own refraction indices.

In the case of modulator such parameters where take into account with using programming language C#:

Specific power of the modulator with band of transitions $\Delta f = 2$ radians and light beam with square cross sections

$$\left(\frac{P}{\Delta f}\right)_2 = 4 \cdot 10^9 \varepsilon_0 \varepsilon \lambda_0^2 ab\Delta\psi^2 / \pi(n')^6 (r')^2 \qquad (14)$$

Deeps of the modulations

$$\eta = \sin^2\left[\frac{\pi^{3/2} n^{7/2} r' \sqrt{lV}}{4S\lambda_0^{3/2}}\right] \qquad (15),$$

except for commonly accepted values, such parameters where used: λ_0 = 0,633 mkm, *a* = 1 mkm, *b* = 1 mkm , *n*′ = 2,2 , *r*′ = $30 \cdot 10^{-12}$ V/m, *r* – electrooptical coefficient for the niobate lithium crystals; $r = 30 \cdot 10^{-12}$ V/m, *l* - length of interaction of optical and electrical energy, *S* – cross-section area of the optical beam.

As the examples of calculation, on the Fig.4 and Fig.5 we can see the dependence of the specific power (mW/MHz) as a function of length of interaction of optical and electrical energy (mkm) for the optical beam with square cross sections with transitions bands Δ*f*=2 radians, and dependence of the specific power (mW/MHz) as a function of overlap parameter (ξ = 0-1) respectively.

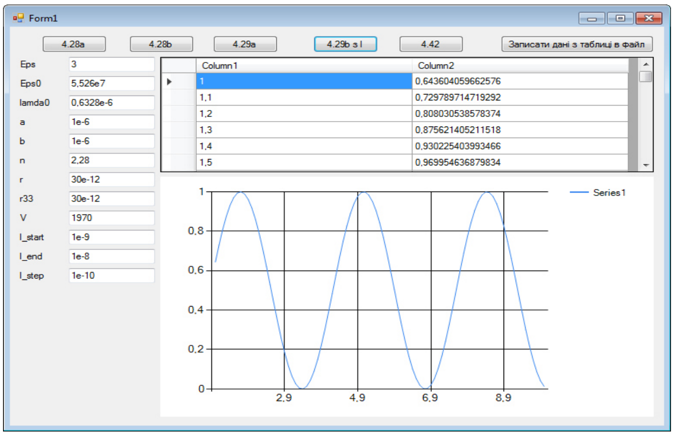

Fig.4. Dependence of the specific power (mW/MHz) as a function of length of interaction of optical and electrical energy (mkm) for the optical beam with square cross sections with transitions bands Δf=2 radians.

III. Discussion and conclusions.

Conducted research has shown the possibility to count basic parameters of the optical waveguides and modulators what will be create on their basis. But for such calculations row of the parameters are needed. Some of them, such as length and thickness are determined fairly easy. The most problem are appearance under the determinations of the refractive indexes. In this case often simulations methods are used. For example, the waveguides may be built from resistive or pure lays GaAs on the n- type substrate from the same materials. As known, the presence of free carriers in the semiconductors is lowering its refractive index in the comparisons with the pure substations. The lays without free carriers will have more hire refractive index, then the substrate with hire concentrations of carriers. When the lowering refractive index of substrate, caused by free carriers, is sufficiently large, then pure lay will be a waveguiding. In the *n*-type GaAs this effect so strong, that allows to create excellent waveguides.

As it was shown in, the decreasing of refractive index mainly caused by the negative depositions of free carries plasma in the dielectric constant. That's why, in calculations the amount of this deposition, instead of free electrons mass, the effective mass *m** of free electrons are used. In the semiconductors with *N* free carriers on the units of volume, the changing of refractive index, that causing by free carriers is equal:

$$n = -\frac{N\lambda_0 q^2}{\varepsilon_0 n_S 8\pi^2 m^* c^2} \tag{16}$$

where n_s – refractive index of semiconductors materials on the wavelength in vacuum λ_0 , q - the charge of free carriers, ε_0- dielectric permittivity of vacuum, c - light velocity. For example, in *n*-type GaAs, Δn_s=0,01, when N=5*10^18^ carries per cm^3, λ_0=1 mkm. Such changing of refractive index on a relation to the *n*-type materials is sufficiently enough for existing waveguide light propagations, when on the *n*-type GaAs substrate can manufactured pure lay of required thickness.

For defining, what waveguide thickness is needed, we must connect this changing of refractive index with requisitions of waveguiding light propagations (5). The modes of GaAs waveguide region that limited from one side by airs, in good approximation can be anti-symmetrical modes with double lay thickness. The refractive index of GaAs is so high, that light energy on the boundary GaAs-air in fact is equal zero. That's why waveguides modes have such cut-off conditions, that for anti-symmetrical modes of symmetrical waveguides.

For the propagations of lightin the waveguides with sickness h, the leap of refractive index Δn between waveguides and substrate must be equal:

$$\Delta n \geq \frac{(2M-1)^2}{2n_S}\left(\frac{\lambda_0}{4h}\right)^2 \tag{17}$$

where M - numbers of anti-symmetrical modes in waveguides.

Combining formulas (16) and (17) available conditions for waveguide cutting in semiconductors waveguide, that where obtained by resistivity layer on the substrate with lower resistance. The necessary difference between the concentrations of carriers in the substrate and waveguides layers is expressed as function of wavegui

$$N_f - N_S > \frac{(2M-1)^2}{4q^2h^2}\cdot \pi^2 \varepsilon_0 m^* c^2 \tag{18}$$

When the concentration of carriers in the film N_f is much less of the concentrations in substrates N_s, the conditions (18) reduce no the condition to the concentration carriers only in substrate. The changing of free carriers concentrations is a convenient preparation method semiconductors material.

In the dielectric materials the changing of refractive index may be calculating in the case of proton bombardment and ion implantation by formula:

$$\Delta n = A \cdot D \tag{19}$$

where D - density of particles, A - appropriate coefficient for using materials. For example, in the case of Li ions with energy 32-200 keV bombarding $A = 2{,}1 \cdot 10^{-21}$.

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